A Probabilistic Approach to Optimal Robust Path Planning with Obstacles

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***Abstract*— Autonomous vehicles need to plan trajectories to a specified goal that avoid obstacles. Previous approaches that used a constrained optimization approach to solve for finite sequences of optimal control inputs have been highly effective. For robust execution, it is essential to take into account the inherent uncertainty in the problem, which arises due to uncertain localization, modeling errors, and disturbances.**

Prior work has handled the case of deterministically bounded uncertainty. We present here an alternative approach that uses a probabilistic representation of uncertainty, and plans the future probabilistic distribution of the vehicle state so that the probability of collision with obstacles is below a specified threshold. This approach has two main advantages; first, uncertainty is often modeled more naturally using a probabilistic representation (for example in the case of uncertain localization); second, by specifying the probability of successful execution, the desired level of conservatism in the plan can be specified in a meaningful manner.

The key idea behind the approach is that the probabilistic obstacle avoidance problem can be expressed as a Disjunctive Linear Program using linear chance constraints. The resulting Disjunctive Linear Program has the same complexity as that corresponding to the deterministic path planning problem with no representation of uncertainty. Hence the resulting problem can be solved using existing, efficient techniques, such that planning with uncertainty requires minimal additional computation. Finally, we present an empirical validation of the new method with a number of aircraft obstacle avoidance scenarios.

I. Introduction

Path planning for autonomous vehicles such as Unmanned Air Vehicles (UAVs) has received a great deal of attention in recent years [1][2][3][4]. UAVs need to be able to plan trajectories that take the aircraft from its current location to a goal, while avoiding obstacles. These trajectories should be optimal with respect to a criterion such as time or fuel consumption.

This problem is challenging for two principal reasons. First, as noted by [3], the optimization problem is inherently non-convex. Second, there are a number of sources of uncertainty in the problem, such as modeling uncertainty, disturbances, and uncertain localization. Planning under uncertainty is a particularly challenging problem that is currently of great interest [5][6].

Previous approaches addressed the first of these challenges; [7] introduced a Mixed-Integer Linear Programming approach that designs fuel-optimal trajectories for vehicles modeled as linear systems. The Mixed-Integer Linear Programming approach uses a Branch and Bound [8] technique to make the non-convex optimization problem tractable. [9] and [3] extended this approach to solve problems in aircraft and spacecraft trajectory planning. By including temporally flexible state plans, [10] was able to generate optimal trajectories for UAVs with time-critical mission plans. [11] showed that using Disjunctive Linear Programming [12] rather than Mixed-Integer Linear Programming, and using *conflicts* [13] to guide the search process leads to more efficiency in solving the optimization problem. As an alternative to these constrained optimization approaches, [14] used a randomized learning-and-query approach known as Probabilistic Roadmaps.

These approaches do not, however, take into consideration uncertainty. This limits the effectiveness of these approaches for applications such as UAVs for three main reasons. First, aircraft location is not usually known exactly, but is estimated using a stochastic system model, inertial sensors and/or a Global Positioning System. In this case, the estimated location of the aircraft is expressed as a distribution such as a Gaussian. Deterministic trajectory design does not take into account the uncertainty in aircraft position. Second, system models are approximations of the true system model, and the system dynamics themselves are usually not fully known. Third, disturbances act on the aircraft that make the true trajectory deviate from the planned trajectory. These existing approaches are not robust to these disturbances, meaning that aircraft can collide with obstacles while executing a plan that was designed to prevent collisions.

On the other hand, uncertainty was handled by [15] and [16] in the trajectory planning problem, for the case of uncertainty with known bounds. In the case of disturbances this corresponds to having a known bound on the magnitude of the disturbance. Robustness is achieved by designing trajectories that guarantee feasibility of the plan as long as disturbances do not exceed these bounds.

Here, we use an alternative approach that characterizes uncertainty in a probabilistic manner and designs control inputs accordingly. In the deterministic optimal trajectory design problem, an optimized sequence of control inputs is designed so that a system passes through a sequence of predicted states that minimize some cost function (for example the time taken to reach a goal state). In the probabilistic path planning problem, we design an optimal sequence of control inputs so that the system passes through a trajectory of state *distributions* that ensure that the probability of successful execution is at least a specified value.

The probabilistic approach to robust path planning with obstacles has a number of advantages over a norm-bounded approach. First, vehicle localization techniques often use a Kalman filter to determine location from inertial sensors or GPS, and these estimate a probabilistic distribution over possible locations. Second, random noise processes are natural models for disturbances caused, for example, by wind, where it may be easier to determine the *expected* value of the disturbance magnitude rather than an absolute upper bound. Finally, by specifying the *probability* that a plan is executed successfully, we can stipulate the desired level of conservatism in the plan in a meaningful manner.

Recent work in probabilistic model predictive control [17] developed methods for designing optimal sequences of control inputs subject to *linear chance constraints*; these ensure that linear constraints are satisfied with a certain probability. We extend this work to show that the problem of probabilistic path planning with obstacles can be expressed as a Disjunctive Linear Program and solved using the same techniques that have been shown to be effective for path planning without uncertainty. The key insight is that the probability of colliding with an obstacle can be upper bounded using a disjunction of linear chance constraints. With our method, these chance constraints are then converted to deterministic linear constraints [18] in order to yield a Disjunctive Linear Program that can be solved using techniques such as those developed by [11], with similar computational complexity as the original path planning problem, without uncertainty.

We demonstrate the method using a number of aircraft trajectory planning scenarios, and present an empirical analysis of the approach. The results show that the method gives a dramatic increase in robustness compared to an approach that does not take into account uncertainty. We show how conservatism can be traded off against fuel efficiency. Finally, we show that the method is conservative, and that the conservatism increases approximately linearly with the number of obstacles in the map.

II. Problem Statement

In this section we define the probabilistic path planning problem as follows:

> *Given a probability distribution for the initial vehicle position, and given a desired goal position, design a finite, optimal sequence of control inputs* $\mathbf{u_0}...\mathbf{u_{k-1}}$ *such that the expected final vehicle position corresponds to the goal position. Take into account uncertainty such that collision with any obstacle at a given time step occurs with at most a probability of* Δ.

Here, optimality can be defined in terms of minimizing fuel consumption or time, for example.

In solving this problem we make two main assumptions. First, the vehicle can be modeled as a linear system. Prior work (for example [7] and [9]) has shown that linear system models can be used to design trajectories for vehicles such as UAVs and satellites. Second, the sources of uncertainty in the problem can be described as additive Gaussian noise with known statistics. This assumption is justified in Section II-A.

A. Sources of Uncertainty

In this work, we consider the case where there is uncertainty in the problem that can be described probabilistically. We consider three sources of uncertainty:

1) The initial position of the vehicle is specified as a probabilistic distribution over possible positions. Vehicle position and dynamic state are typically estimated from inertial sensors or global positioning data, and hence are not known exactly. Instead estimation techniques such as Kalman filtering or least squares estimation specify a Gaussian distribution over system state. In this work we assume that the initial position of the aircraft is specified as a Gaussian distribution, and that a Gaussian filtering technique such as a Kalman Filter is used to estimate the location of the aircraft on-line.
2) The system model is not known exactly. Uncertainty in the system model may arise due to modeling errors or linearization. As in many stochastic estimation and control techniques this uncertainty may be modeled as a Gaussian white noise process added to the system dynamic equations [19].
3) Disturbances act on the vehicle. These are modeled as an additional Gaussian noise process added to the system dynamics. In the case of an aircraft, this process represents accelerations caused by wind. The standard deviation of this Gaussian process is the expected absolute value of the disturbance acceleration.

We assume a linear, discrete-time system model in a similar manner to [7]:

$$\mathbf{x}_{t+1} = A\mathbf{x}_t + B\mathbf{u}_t + \omega_t + \nu_t. \tag{1}$$

Here, ω is a Gaussian white noise process that represents model uncertainty, and is distributed according to $\omega_t \sim \mathcal{N}(0, Q)$. The Gaussian white noise process ν models disturbances, and is distributed according to $\nu_t \sim \mathcal{N}(0, R)$. The assumption of zero mean, white noise is made for modeling simplicity; the methods described in this paper apply equally to colored, non-zero mean noise as long as the statistics are known.

III. Probabilistic Path planning as Disjunctive Linear Programming

In this section we show that the probabilistic path planning problem described in Section II can be posed as a Disjunctive Linear Program. The key insight is that the obstacle avoidance problem can be reformulated using chance constraints, and then these chance constraints can be expressed as linear constraints, giving rise to a disjunctive linear program.

A. Probabilistic Path Planning

In deterministic path planning, we predict the future state of a vehicle given a particular sequence of control inputs. This prediction, evaluated against some optimality criterion, is used to optimize the control inputs. In order to plan a path for a vehicle under uncertainty, we must be able to predict the future *distribution* of the vehicle's position, given a particular sequence of control inputs [17].

In Section II we assumed that the initial state has a Gaussian distribution $\mathcal{N}(\hat{\mathbf{x}}_{\mathbf{0}}, P_0)$, that the system dynamics are linear, and that there are additive Gaussian white noise processes corresponding to model uncertainty and disturbances. Under these assumptions, the distribution of the future state is also Gaussian, i.e. $p(X_t|\mathbf{u}_{0,\ldots,k-1}) \sim \mathcal{N}(\mu_t, \Sigma_t)$. By recursive application of the system equations the distribution of the future state can be calculated exactly as:

$$\mu_t = \sum_{i=0}^{t-1} A^{t-i-1} B\mathbf{u}_i + A^t \hat{\mathbf{x}}_{\mathbf{0}} \quad (2)$$

$$\Sigma_t = \sum_{i=0}^{t-1} A^{t-i-1} Q (A^T)^{t-i-1} + \sum_{i=0}^{t-1} A^{t-i-1} R (A^T)^{t-i-1} + A^t P_0 (A^T)^t \quad (3)$$

There are two important properties to note here:

1) The equation for the mean of the state at time t is linear in the control inputs $\mathbf{u}_0, \ldots, \mathbf{u}_{t-1}$
2) The covariance of the state at time t is not a function of the the control inputs $\mathbf{u}_0, \ldots, \mathbf{u}_{t-1}$. This means that for a given initial state covariance, and with known noise covariances, the covariance at a future time is known exactly.

These two properties enable the obstacle avoidance problem to be framed as a Disjunctive Linear Program, as will be shown in Section III-D.

B. Obstacle Avoidance using Linear Chance Constraints

In this section we show that the probabilistic obstacle avoidance criterion in Section II can be expressed conservatively using linear chance constraints.

Chance constraints were previously used by [20] in robot trajectory planning without obstacles, where the uncertainty was due to unknown system parameters; chance constraints were used to prevent the joint angles and joint velocities from going outside allowable limits. [17] used linear chance constraints for finite horizon control design, and showed that uncertainty due to state estimation, disturbances and modeling errors can be handled in one unified framework. We extend this work to path planning with obstacles.

A convex polyhedral obstacle, such as that shown in Fig. 1 is defined using N straight-line segments. The vehicle collides with the obstacle at time step t if its position is within the obstacle, in other words if (4) is satisfied.

$$\bigwedge_{i=1,\ldots,N} \mathbf{a}_i^T \mathbf{x}_t < b_i. \quad (4)$$

Hence the condition defining collision with a given obstacle is a conjunction of linear constraints on the position of the vehicle.

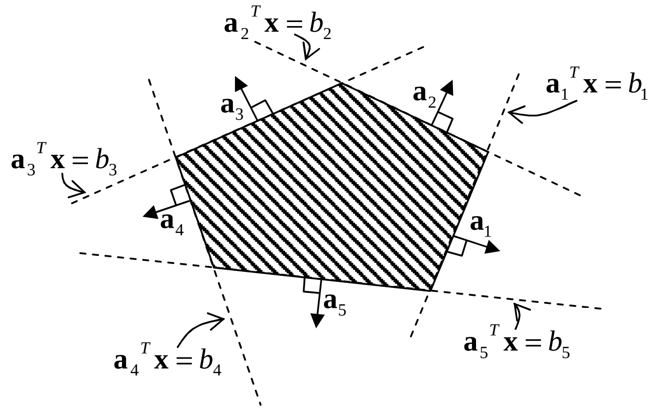

Fig. 1. Two-dimensional obstacle modeled as a convex polyhedron. The vectors $\mathbf{a}_1, \ldots, \mathbf{a}_N$ are the unit outward normals to the N line segments that define the obstacle.

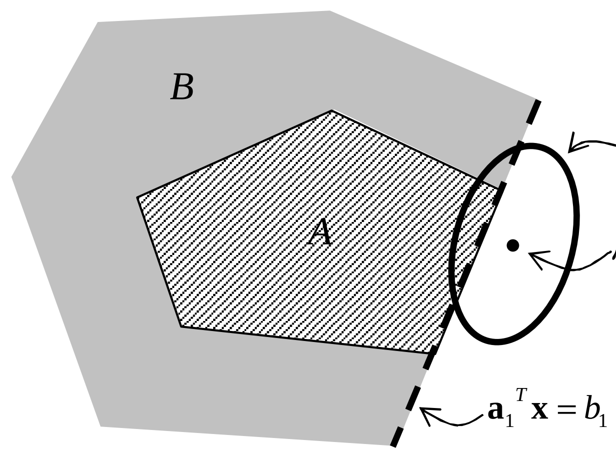

Fig. 2. Predicted distribution of X_t, shown as mean μ_t and covariance ellipse Σ_t, in relation to obstacle. The probability of satisfying the constraint $\mathbf{a}_i^T X_t < b_i$ is the integral of this distribution over the region $A \cup B$, which is greater than or equal to the integral over the region A, the probability of collision.

In the probabilistic path planning problem, the future position of the vehicle at time t is a random variable, which we denote X_t. Under the assumptions in Section II, X_t is a Gaussian defined by a mean μ_t and covariance Σ_t. The key insight to posing obstacle avoidance using chance constraints is that the probability of *any* of the linear constraints in (4) being satisfied is an *upper bound* on the probability of the vehicle colliding with the obstacle. This is illustrated in Fig. 2.

In general, for a convex polyhedral obstacle defined by N line segments,

$$i = 1, \ldots, N \qquad p(collision) \le p(\mathbf{a}_i^T X_t < b_i) \quad (5)$$

Hence the requirement:

Probability of collision with a given obstacle at a given time step t is less than or equal to δ,

can be expressed conservatively as a disjunction of linear chance constraints on the position of the vehicle at time step t:

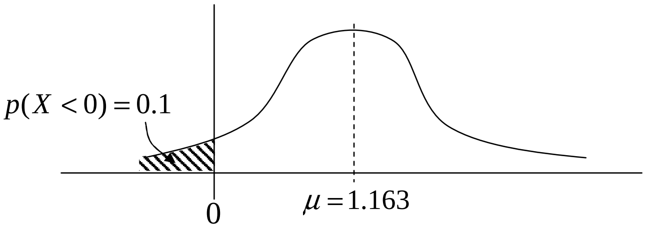

Fig. 3. Univariate Gaussian distribution with mean μ and variance 1. For fixed variance, the chance constraint $p(X < 0) \leq 0.1$ is satisfied if and only if $\mu \geq 1.163$

$$\bigvee_{i=1,\ldots,N} p(\mathbf{a}_i^T X_t < b_i) \leq \delta. \tag{6}$$

Note that for non-convex obstacles, the inequality in (5) does not apply, and hence we restrict our attention to convex polyhedral obstacles.

C. Linear Chance Constraints as Deterministic Linear Constraints

In this section, we show that linear chance constraints on the position of the vehicle at time t, can be expressed as deterministic linear constraints on the mean of the vehicle position at time t [18].

First, consider the univariate Gaussian random variable X shown in Fig. 3, which has mean μ and variance $\sigma^2 = 1$, and the chance constraint $p(X < 0) \leq 0.1$. In Fig. 3, $p(X < 0)$ is exactly 0.1 for $\mu = 1.163$. Note that, if the variance is fixed, $p(X < 0) \leq 0.1$ if and only if $\mu \geq 1.163$. Hence the chance constraint on the random variable X, $p(X < 0) \leq 0.1$, can be translated into a deterministic constraint on the mean μ if the variance σ^2 is known. In this case, for a variance of one, the corresponding deterministic constraint is $\mu > 1.163$.

In general, a chance constraint on a singlevariate Gaussian random variable $X \sim \mathcal{N}(\mu, \sigma^2)$ with fixed variance but variable mean, can be translated into a deterministic constraint on the mean:

$$p(X < 0) \leq \delta \Longleftrightarrow \mu \geq c \tag{7}$$

The value of the deterministic constraint c is calculated as follows:

$$c = \sqrt{2}\sigma \cdot erf^{-1}(1 - 2\delta), \tag{8}$$

where erf is defined as:

$$erf(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt. \tag{9}$$

The inverse of erf can be calculated using a look-up method. Note that only one look-up table is required for any Gaussian distribution. For (8) to be valid, we assume that the probability δ is less than 0.5.

Now consider the case of a multivariate Gaussian random variable X_t, corresponding to the position of the vehicle at time t, which has mean μ_t and covariance Σ_t, and the linear chance constraint $p(\mathbf{a}^T X_t < b) \leq \delta$. The event $\mathbf{a}^T X_t < b$ is

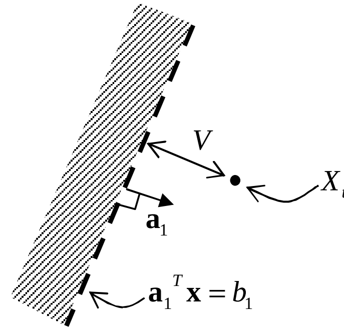

Fig. 4. Linear constraint and vehicle position X_t. V is the distance between the constraint and the vehicle, defined as positive for values of X_t for which the constraint is satisfied, and negative for value of X_t for which the constraint is violated. The vector $\mathbf{a}$ is the unit normal in the direction of positive V.

equivalent to the event $V < 0$, where V is the perpendicular distance between the constraint $\mathbf{a}^T\mathbf{x} = b$ and $\mathbf{x}$, as shown in Fig. 4.

The random variable V is a derived variable of the multivariate random variable X_t. It can be shown that V is a univariate Gaussian random variable, with mean μ_v and variance σ_v, where:

$$\mu_v = \mathbf{a}^T \mu_t - b, \tag{10}$$

and

$$\sigma_v = \sqrt{\mathbf{a}^T \Sigma_t \mathbf{a}}. \tag{11}$$

The linear chance constraint $p(\mathbf{a}^T X_t < b) \leq \delta$ is therefore equivalent to a chance constraint $p(V < 0) \leq \delta$ on the univariate Gaussian random variable V. As described previously in this section, this can be expressed as a deterministic constraint on the mean, of the form $\mu_v \geq c$, where c is given by (8), with $\sigma = \sigma_v$.

Expressing this deterministic constraint in terms of the original variable X_t yields:

$$p(\mathbf{a}^T X_t < b) \leq \delta \Longleftrightarrow \mathbf{a}^T \mu_t - b > c, \tag{12}$$

where:

$$c = \sqrt{2\mathbf{a}^T \Sigma_t \mathbf{a}} \cdot erf^{-1}(1 - 2\delta). \tag{13}$$

This calculation requires knowledge of Σ_t, the covariance of the state at time t. In Section III-A we showed that Σ_t does not depend on the control inputs, and therefore given an initial state covariance and the noise process covariances, we can calculate Σ_t using (3).

Hence linear chance constraints on the position X_t of the vehicle at time t, can be expressed as deterministic linear constraints on the expected position, μ_t, of the vehicle at time t. This means that the disjunction of linear chance constraints (6) can be expressed as a disjunction of deterministic linear constraints on the mean of the vehicle position:

$$\bigvee_{i=1,\ldots,N} \mathbf{a}_i^T \mu_t - b_i > c_i. \tag{14}$$

D. Handling Multiple Obstacles

The previous sections showed that the requirement:

Vehicle collides with a given obstacle A at time t with probability at most δ,

can be expressed as a disjunction of deterministic linear constraints on the mean of the vehicle position at time t. The problem statement in Section II, however, requires that the vehicle collide with *any* obstacle at time t with probability of at most Δ.

In general, for two events A and B:

$$p(A \cup B) \leq p(A) + p(B), \tag{15}$$

Therefore we specify the following constraint for all obstacles $O_i \in \{O_1, O_2, \ldots, O_M\}$ and for all time steps $t_j \in \{t_1, \ldots, t_k\}$ in the planning horizon:

Collide with obstacle O_i at time t_j with probability at most $\frac{\Delta}{M}$.

Let C be the event that collision occurs with any obstacle time step t in the planning horizon, and D_i be the event that collision occurs with obstacle O_i at time t. Then following from the new constraint and (15),

$$p(C) \leq \sum_{i=1}^{M} p(D_i) \leq \sum_{i=1}^{M} \frac{\Delta}{M} = \Delta \tag{16}$$

as required.

Note that a similar method could be used to handle the case where the problem statement requires the probability of collision *over the entire planning horizon* to be constrained, by dividing Δ by the number of time steps in the horizon.

IV. DLP Summary

The problem formulation in Section II therefore gives rise to the following constraints on the mean and covariance of the distributions of X_0 to X_k:

1) Goal requirement: The expectation of X_k must be the goal $\mathbf{g}$.
$$\mu_k = \mathbf{g} \tag{17}$$
2) Obstacle avoidance: For each time step $t = 1, \ldots, k$ and for each obstacle $j = 1, \ldots, M$,
$$\bigvee_{i=1,\ldots,N_j} \mathbf{a}_{ij}^T \mu_t - b_{ij} > c_{ij}. \tag{18}$$
Here, each obstacle j is defined by N_j constraints of the form $\mathbf{a}_{ij}^T \mathbf{x} < b_{ij}$. The value c_{ij} is calculated as in (13) with $\delta = \frac{\Delta}{M}$.

We now show that these constraints are linear in the control inputs $\mathbf{u_0}, \ldots, \mathbf{u_{k-1}}$. First, we address the goal requirement. Using (2) to calculate the mean of the final state, it follows that:

$$\mu_k = \sum_{i=0}^{k-1} A^{k-i-1} B \mathbf{u}_i + A^k \hat{\mathbf{x}}_\mathbf{0}. \tag{19}$$

Since this is linear in the control inputs, the constraint $\mu_k = \mathbf{g}$ is linear in the control inputs.

The obstacle avoidance requirement gives a conjunction of disjunctions of linear inequality constraints on μ_t for $t = 1 \ldots k$, as shown in (18). Note that the value c_{ij} does not depend on the control inputs. The predicted means μ_t are linear in the control inputs, as shown in (2). Hence the obstacle avoidance requirement is linear in the control inputs.

Finally, both the minimum time and minimum fuel optimality criteria can be expressed linearly. The minimum fuel criterion, as described by [3], can be used without modification. The minimum time criterion described by [9] can be modified so that the time for the *expected* vehicle position to reach the goal is minimized. From (19), the criterion remains linear in the control inputs.

The robust probabilistic path planning problem with obstacles can therefore be expressed as a Disjunctive Linear Program, and solved efficiently using existing methods [11]. The additional computational complexity required to handle uncertainty compared to the original, deterministic path planning problem, is small, consisting only of a single look-up table evaluation per constraint of the original problem.

V. Experimental Results

The new method for path planning with obstacles was implemented and tested using an aircraft path planning scenario, where minimum-fuel robust trajectories were to be designed over a fixed planning horizon. Results were obtained for several different obstacle maps. This section shows the key results from these tests.

First we show trajectories generated by the new method, and show how these vary as the specified maximum probability of collision, Δ, is varied. We compare these trajectories to one generated without taking into account uncertainty. Second, we compare the true probability of collision for the trajectories generated by the new method, to the specified maximum probability of collision, Δ. We demonstrate the robustness of trajectories designed using the new method compared to the optimal path that does not take into account uncertainty. The results show that the method has a relatively high level of conservatism. We show how this conservatism varies with the number of obstacles in the map. The reasons for this are discussed.

The aircraft was modeled as a point mass subject to minimum and maximum velocity constraints, as well as maximum acceleration constraints [3][7][10]. The disturbance acceleration had an expected absolute value of $0.01m/s^2$ in both the x and y directions. For comparison, the maximum commandable acceleration magnitude for the aircraft was $0.5m/s^2$. The variance of the initial state was $0.03m^2$ in both the x and y directions. The modeling error was described by additive zero mean white noise processes on the vehicle velocity in both the x and y directions, which had a standard deviation of $0.01m/s$.

A. Trajectories Generated by Probabilistic Method

Fig. 5 shows the trajectories planned by the new probabilistic method for three different values of the maximum

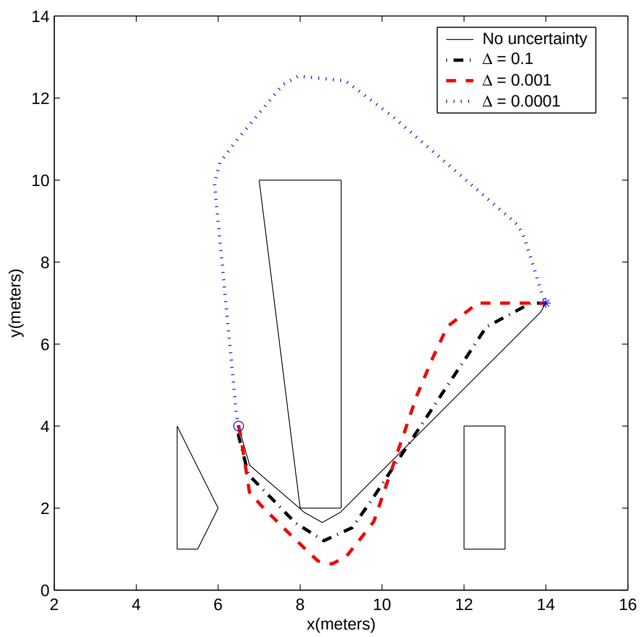

Fig. 5. Trajectories planned by new probabilistic method. The circle is the mean of the start distribution, while the star is the goal position. The different trajectories represent different maximum probabilities of collision Δ. For $\Delta \leq 0.0001$, the planned path is no longer between the obstacles, but instead goes around the large obstacle. This solution requires more fuel, but has a lower probability of collision. The fuel-optimal path planned without taking into account uncertainty is shown for comparison.

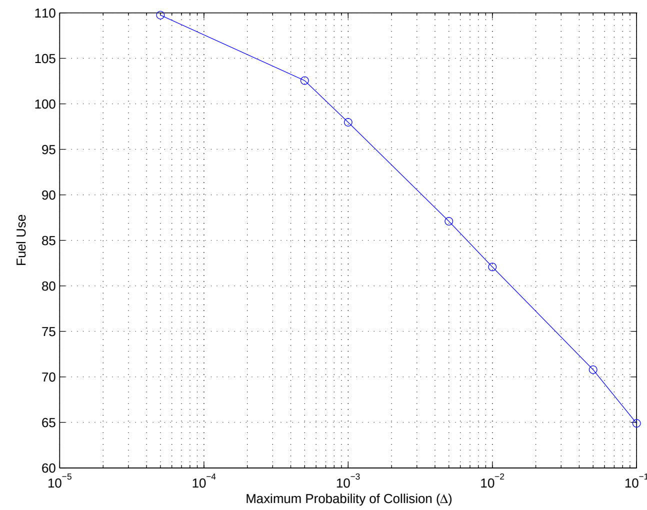

Fig. 6. Fuel use as a function of maximum collision probability Δ, for the map shown in Fig. 5. The fuel use decreases as Δ increases, since the level of conservatism decreases.

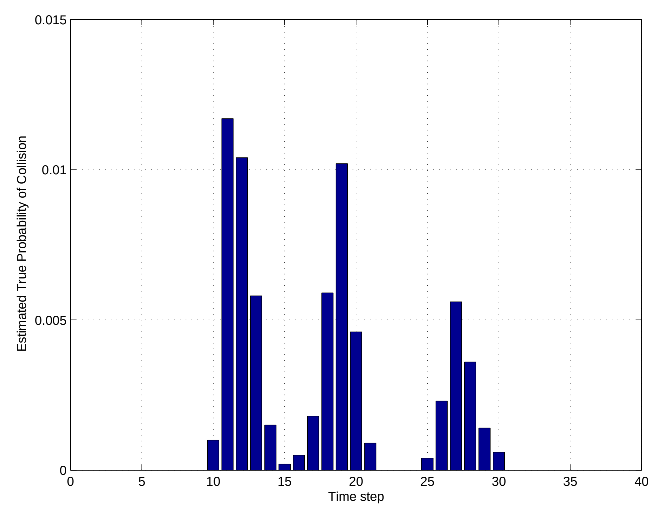

Fig. 7. Estimated true probability of collision for each time step in a typical plan where $\Delta = 0.1$. The highest probability of collision is still signifi cantly lower than Δ, indicating signifi cant conservatism in the plan.

probability of collision Δ for a typical obstacle map. As the probability of collision decreases, the planned trajectory moves further away from the obstacles. For a probability of collision less than 0.0001, the planned trajectory can no longer go between the obstacles, but instead must go around the largest obstacle.

Fig. 6 shows how the fuel use varies with the maximum collision probability Δ. Decreasing Δ increases the level of conservatism in the plan, which causes the fuel use to increase. This demonstrates how conservatism can be traded off against fuel use.

B. Analysis of True Failure Probability

By carrying out a large number of simulations using the planned trajectories, an approximate value for the true probability of collision for each time step in a scenario was calculated. Fig. 7 shows the probability of collision for all of the time steps in a typical plan, where $\Delta = 0.1$. The probability of collision is below the maximum level Δ, as required.

In Fig. 7 the probability of collision is higher for some time steps than others. This is because the chance constraints are not active at all time steps. In order to assess the level of conservatism in the plan, we compare the highest probability of collision, where the chance constraints are tight, to the specified probability Δ. The method is conservative, since the highest probability of collision p_{max} is significantly below the maximum level Δ. We define the conservatism factor as $\frac{\Delta - p_{max}}{p_{max}}$. A factor of zero indicates no conservatism.

Fig. 8 shows how the conservatism factor varies with the number of obstacles in the map. With only one obstacle, the conservatism factor is close to zero, while the factor increases approximately linearly with the number of obstacles. It is clear therefore, that the majority of the conservatism in the problem arises from the bounding used in (16). Ongoing work aims to reduce this conservatism using an iterative approach.

For the sake of comparison, the optimal trajectory for the map shown in Fig. 5 was generated without taking into account uncertainty. While the fuel use for this trajectory is only 48.6, compared to the values shown in Fig. 6 that range between 65 and 110, the probability of collision for a given time step was as high as 0.4. Hence the new probabilistic method can lead to significant robustness gains. These robustness gains are achieved with only a marginal

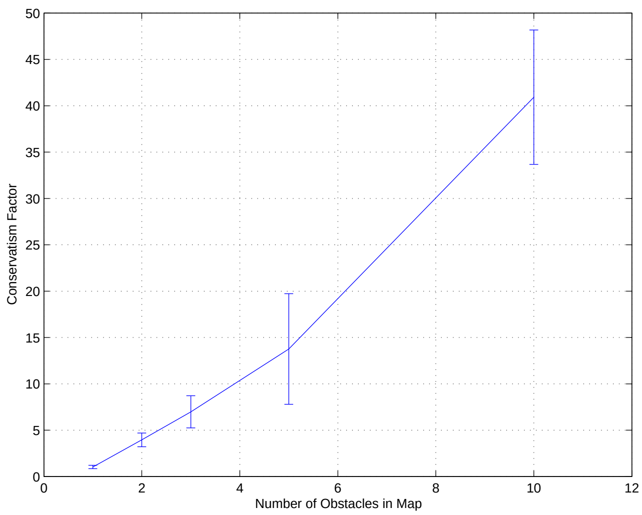

Fig. 8. Conservatism factor as a function of the number of obstacles, averaged over several different arrangements of obstacles. The error bars represent two standard deviation intervals. For one obstacle, the factor is close to zero, but increases approximately linearly with the number of obstacles in the map.

increase in computational complexity.

VI. Conclusion

This paper presents a new, probabilistic approach to optimal, robust path planning with obstacles. We specify a maximum probability that the vehicle collides with any obstacle. We show how this problem can be posed conservatively as a Disjunctive Linear Program, and can therefore use existing constrained optimization methods to generate a finite sequence of optimal control inputs. The resulting optimization problem has the same complexity as the path planning problem that does not take into account uncertainty.

Experimental results showed how the planned path changes with the level of conservatism, specified using the probability of collision. The results showed that the method is conservative, and that the main source of conservatism is due to the bound necessary to accommodate several obstacles. While conservatism is undesirable, the computational simplicity of the approach is highly attractive. Future work will use an iterative approach to reduce the level of conservatism.

While the method presented here was demonstrated in the case of a fixed planning horizon, it can equally be used within a receding horizon framework. Uncertainty about the future state of the vehicle always grows as the distribution is predicted further into the future. This means that the planned path of the mean is further away from obstacles later in the path. In a receding horizon framework, an estimation scheme determines the distribution of the vehicle state on-line, given the most recent observations. As new observations are made, uncertainty about the vehicle state typically diminishes. Hence the path can be planned with less conservatism, than that planned without closing the estimation loop.

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